High space bandwidth product computer-generated holograms using volume holography

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ABSTRACT

We suggest a method for coding high resolution computer-generated volume holograms. It involves splitting the computer-generated hologram into multiple holograms, each individually recorded as a volume hologram utilizing the maximal resolution available from the spatial light modulator. Our method enables their simultaneous subsequent reconstruction. We demonstrate the recording and the reconstruction of a computer-generated volume hologram with a space bandwidth product much higher than the maximal one of the spatial light modulator used as an interface. Finally, we analyze the scheduling procedure of the multiple holographic recording process in photorefractive medium in this specific application.

1. INTRODUCTION

Binary computer-generated holograms (CGHs) are becoming essential components in optical signal processing schemes. Apart from their use as

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a means for storage and reconstruction of images, these holograms can serve as spatial filters, optical elements and as the basic component in interconnection networks. CGHs are created in an electronic computer, and displayed on spatial light modulators (SLMs). This provides real-time variability, which enhances the flexibility of the holograms, and makes them suitable for applications such as adaptive optics and reconfigurable interconnects.

Unfortunately, most currently available SLMs suffer from a limited information capacity, low resolution, distortions, non-uniformities and inter-pixel dead areas. This substantially deteriorates the space bandwidth product (SBP) of the CGHs, and results in a undesired difference between the original image and the one actually reconstructed from a CGH.

In this Letter we demonstrate a method for increasing the SBP of a binary CGH much above the maximal resolution available from the SLM. It includes splitting the CGH into secondary holograms, each utilizing the maximal resolution available from the SLM. The individual secondary holograms are sequentially transmitted from the computer to the SLM, and imaged onto a volume holographic medium. They are then converted to volume holograms, all recorded with the same reference wave, and stored in the volume of the holographic medium. The subsequent readout results in the reconstruction of the whole original high resolution image, with a large reduction in the reconstruction error. In this vein, we have used a photorefractive (PR) crystal as the storage medium, and also utilize its coupling properties for significantly improving the recording efficiency of the multiple volume (secondary) holograms.

2. ERROR REDUCTION BY USING COMPUTER-GENERATED VOLUME HOLOGRAPHY

Consider a binary CGH expressed by the spatial distribution (in the plane of the SLM) of H(u,v), which is the Fourier transform of h(x,y). The hologram is designed to reconstruct the image f(x,y) in a sub-area A in the x-y plane. The reconstruction error (per pixel in A) is defined as:

$$e=\frac{1}{A}\int_A |f(x,y)-\beta\hat{h}(x,y)|^2dxdy \qquad (1)$$

where $\hat{h}(x,y)$ is the part of $h(x,y)$ included in the sub-area A, and β is a constant which is designed to minimize the reconstruction error[1]. We show that the reconstruction error of a conventional binary CGH decreases when the area of the reconstructed image becomes smaller. In fact, the error should theoretically completely disappear when the image is a single point. Based on this observation, we analyze a general case of a complicated image, and desire to code its Fourier transform by the CGH. We decompose the image into simple primitives of a small area, and calculate a CGH for each one of them. The whole set of CGHs is sequentially transmitted to an SLM, and imaged onto a PR crystal. The CGHs are converted to volume holograms by the method we introduced in a previous work[2], and stored in the crystal as thick gratings. The volume holograms are formed due to interference between a nonzero diffraction order (off the SLM) which bears the holographic data, and the zero (non-diffracted) order. The resultant thick gratings differ from each other by their periodicity and orientation, and may be addressed (reconstructed) individually as separate data pages[2], or as a one, high resolution, image. Here we utilize the last option and reconstruct the original high SBP image, by illuminating the crystal with a readout beam directed along the optical axis. In this method, the reconstruction error of the full image is the average of all the errors of the primitives. Therefore, as we choose smaller primitives the overall error is reduced. The maximal number of primitives that can be used in this technique is limited only by the scheduling procedure of the holographic recording in the PR crystal[3]. As will be shown below, the maximal number of stored holograms can be larger, in this special configuration, than in the usual recording procedures.

The classic methods of coding a complex function $F(u,v)$ on a binary CGH (the detour coding methods[4]) are based on replacing every analog value of $F(u,v)$ by a binary valued matrix. As the matrix becomes larger, the original values are described more precisely, and the reconstruction error decreases. Most currently available SLMs, however, contain only a small amount of binary pixels, and this limits the resolution $F(u,v)$. On a SLM of 128×128 pixels, for example, only a binary CGH of a function of 8×8 can be displayed, making the commonly accepted[5] assumption that every original value is expressed by a binary 16×16 matrix. The immediate conclusion is that, wherever the SLM possesses a low resolution, one has to use another method. In any case, one should expect in such instance a significant reconstruction error for coding high dynamic range functions. In the present work, we choose to design the CGHs by the Projection Onto Constraint Set (POCS) algorithm[1], since it is fast and yields relatively small reconstruction errors. The constraints in the image plane are to obtain (I) a desired complex amplitude distribution at an a priori defined sub-area, and (II) its conjugate opposite at a region that corresponds to the reflection of that sub-area with respect to the origin. The constraint in the CGH plane is the binarization of the hologram's values. As mentioned above, an image of a small area can significantly reduce the reconstruction error, while larger images include larger errors. To demonstrate this principle we calculate the reconstruction error from two binary CGHs that were designed using the POCS algorithm. The CGHs were of two images that extend over different areas: the images of the letters "R" and "O". Figure 1 shows the reconstruction error e (according to Eq. 1) versus the image area (number of pixels).

3. EXPERIMENTAL RESULTS

Our experimental setup is sketched in Fig. 2. Each binary CGH is displayed on the SLM, coherently imaged onto a PR crystal, and converted to a volume hologram[2]. In this process all the CGHs are recorded with the same reference wave: the zero-th order of diffraction from the SLM. Therefore, illuminating with a readout beam along the optical axis (which propagates counter to the zero order) results in Bragg matching to all the stored holograms, and reconstructs all the primitives simultaneously (and hence the whole high resolution image).

We defined the smallest primitive (or the most delicate partition) of a complicated image as a point, and recorded a volume CGH for each point that composed the image. Since a binary CGH of a point is merely one rectangular grating (owing to the binary nature of the SLM), and bears no additional information, we expect to have the minimal reconstruction error for each one of them. The relative intensity of each reconstructed point can be controlled either by the scheduling in the recording process of the multiple holograms, or by the angular response of the PR crystal. Its phase is obtained by the proper lateral shift of the grating on the SLM. The minor reconstruction errors using this method originate from small non-uniformities in the PR crystal and aberrations of the optical system, rather than from the lack of resolution in the

SLM as in planar CGHs. Ideally, every binary grating can be reconstructed into a desired point with no error at all. Moreover, since the reminiscent errors are not fundamentally built-in in the coding method, the recorded CGHs can be iteratively corrected[6], and all the errors can, in principle, be eliminated.

Our experimental results are depicted in Fig. 3, where the reconstruction of the amplitude distribution, with a constant phase, of a delicate (made of individual dots) letter "R" was performed with (a) one CGH only (error identical to a "conventional" planar CGH case), (b) 6 CGHs, each designed to reconstruct one primitive line of the letter, and (c) 16 CGHs, one for each point in the "R". The improvement in the reconstruction quality is clearly seen.

4. THE SCHEDULING PROCEDURE OF THE RECORDING PROCESS

The scheduling of the recording process of the multiple holograms in the PR crystal requires a special attention. Previously suggested schemes for scheduled[3,7,8] and incremental[9] multiplexing conclude that the diffraction efficiency of one hologram, when N volume holograms are stored, decreases with $1/N^2$. This is due to the fact that when one hologram is recorded, all previously stored ones are not Bragg matched to the interfering beams, and hence experience erasure. In that sense, the erasure process takes place as if the erasing beams were incoherent or some background illumination. Our recording configuration differs from those techniques since all the holograms are recorded with the same reference, which is therefore Bragg-matched to all of them. Consequently, during a sequential recording process of one hologram after the other, the reference beam continues to deflect light in the directions of the previously recorded gratings, at any recording time interval. This slows down dramatically the erasure process, and induces a large asymmetry between the writing and the erasure times. A generic solution in the non-depleted-pumps approximation for erasures with one Bragg-matched beam were discussed by Horowitz et al[10]. We use this idea to increase the number of stored holograms while maintaining a relatively high diffraction efficiency.

We demonstrate the principle of scheduling the recording process of multiple holograms stored with the same reference, by comparing the perturbation in the refractive index versus the number of holograms (N) in two cases: (I) when the coupling between the reference and the image-bearing beam is significant (so-called "our case") and (II) when there is no coupling between the recording beams. The last case is also the case described in Refs. 8 and 9, since, in the absence of coupling, the use of a common reference beam does not affect the resultant diffraction efficiency and the temporal response of recording process (experimentally, recording with uncoupled beams is possible, as will be described below, by utilizing the polarization dependence of the coupling coefficient). The equations that describe the time-dependent wave-mixing process[11] are

$$\frac{\partial A_i}{\partial z}=\frac{ik}{n}\Delta n_i A_R \tag{2}$$

$$\frac{\partial A_R}{\partial z}=\frac{ik}{n}\sum_{i=1}^{N}\Delta n_i^* A_i \tag{3}$$

$$\frac{\partial \Delta n_i}{\partial t}+I_0(t)\,\Delta n_i=\gamma_i A_i A_R^* \tag{4}$$

where $A_i(z,t)$ is the field amplitude of the image-bearing beam i, i=1,2,..,N. For this calculation we assume that each band-limited beam A_i is represented by a single plane-wave, and all A_i's are equal. Also, $A_R(z,t)$ denotes the reference beam. The overall light intensity in a time-interval during the recording process is given by $I_0(t)=|A_R(t)|^2+|A_i(t)|^2$. Note, that $I_0(t)$ is constant for every z plane, and we make the standard low visibility assumption (commonly used in describing the PR effect) that $A_R A_i^* \ll I_0$. The i-th PR perturbations in the refractive index n is $\Delta n_i(z,t)$, γ_i is the PR coupling coefficients and k is the light wave-number in vacuum. We assumed negligible absorption (which does not affect the qualitative process in the transmission geometry anyway), and all units are given in equivalent dark irradiance units (we neglect[11] the dark current). The boundary conditions for recording process are $A_i(0,t)$, $A_R(0,t)$ and $\Delta n_i(0,t)$ (for i=1..N), where the input fields alternate in time in the incremental recording procedure[9], and the initial conditions are $\Delta n_i(z,0)=0$ for all i, and the input amplitudes $A_i(0,t)$ may be varied to optimize the recording process. Note that besides the "useful" gratings Δn_i's, another set of gratings may form due to interference between the pairs A_i and A_j (for all i,j), even though the input fields $A_i(0,t)$ and $A_j(0,t)$ do not overlap in time.

We have performed a preliminary check on the theoretical results, by comparing, in both cases, the maximal number of holograms to be recorded that yielded a given diffraction efficiency. When we used ordinary polarized beams in the recording process in $BaTiO_3$, the beams did not couple and we were able to store not more than 20 holograms with a diffraction efficiency of about 1% per hologram. When we used extraordinary polarization we successfully stored more than 50 holograms, with the same diffraction efficiency threshold. This qualitatively proves our conclusions. However, since good Bragg matching is required in the readout process, the use of identical polarizations for both the recording and the readout process is highly desirable. Otherwise, the difference in the refractive indices in this anisotropic crystal deteriorates the quality of the high-resolution reconstructed image. Therefore, for high density data storage, as well as in the application presented here, the recording beams always couple to each other. Therefore a more general comparison between the models for coupled and uncoupled beams in the recording process is required. Since this extends beyond the scope of the issues presented here, we provide a complete treatment for coupled recording beams elsewhere.

5. CONCLUSIONS

In conclusion, we have demonstrated a new technique to code high SBP computer-generated holograms, at a resolution which greatly exceeds the limitations imposed by the interfacing spatial light modulator. We note that this is the first (to the best of our knowledge) successful attempt of computer controlling information stored in a volume. Our method is useful for coding masks, filters, interconnects and optical elements for applications where high resolution is required but storing large amounts of data is not necessary. Besides signal processing purposes, it can be used as a network of reconfigurable interconnects for computation tasks, and even for coding three-dimensional images (for example, the points in Fig. 3c can be of minimal sizes in different planes of reconstruction).

6. ACKNOWLEDGMENTS

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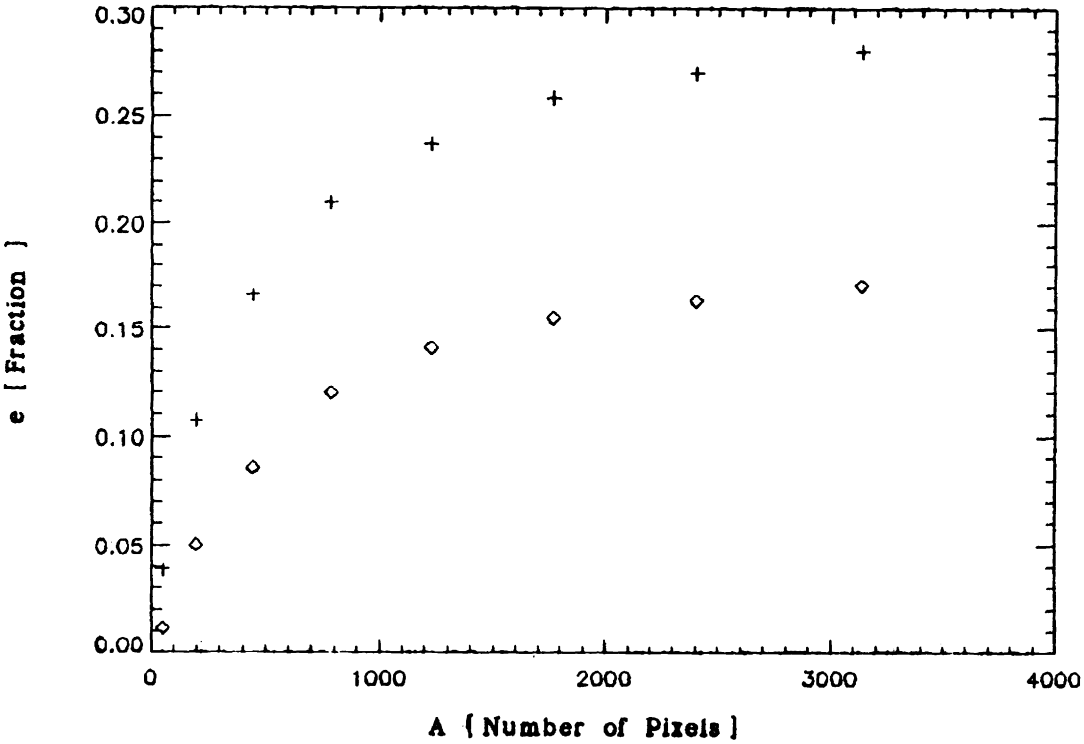

Figure 1: The reconstruction error e as a function of the reconstructed image area A, for images of the letters "R" (+) and "O" (◊).

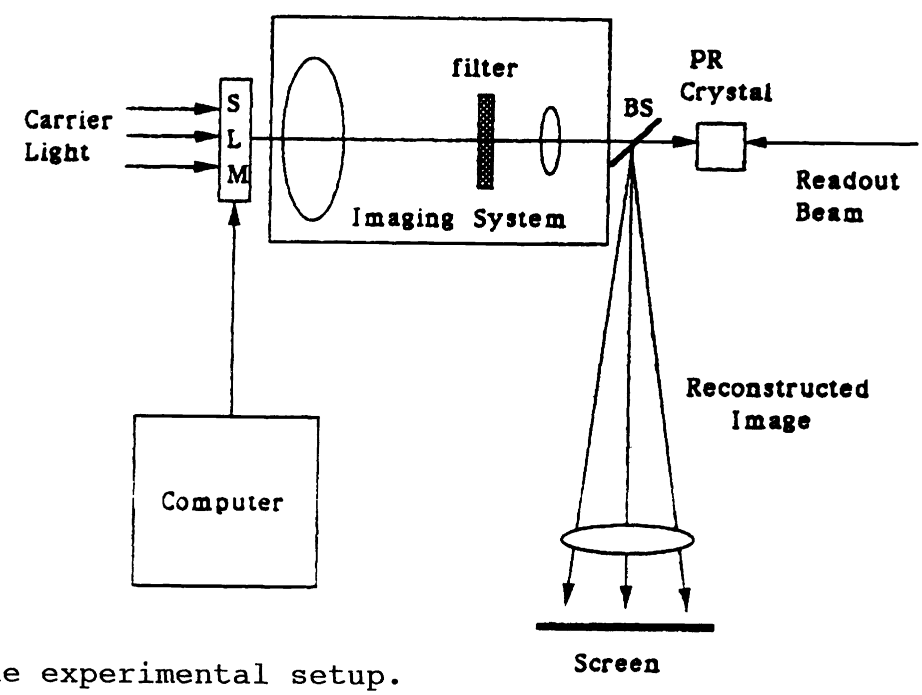

Figure 2: The experimental setup.

Figure 3: The reconstructed image from one (a) six (b) and sixteen (c) CGHs.

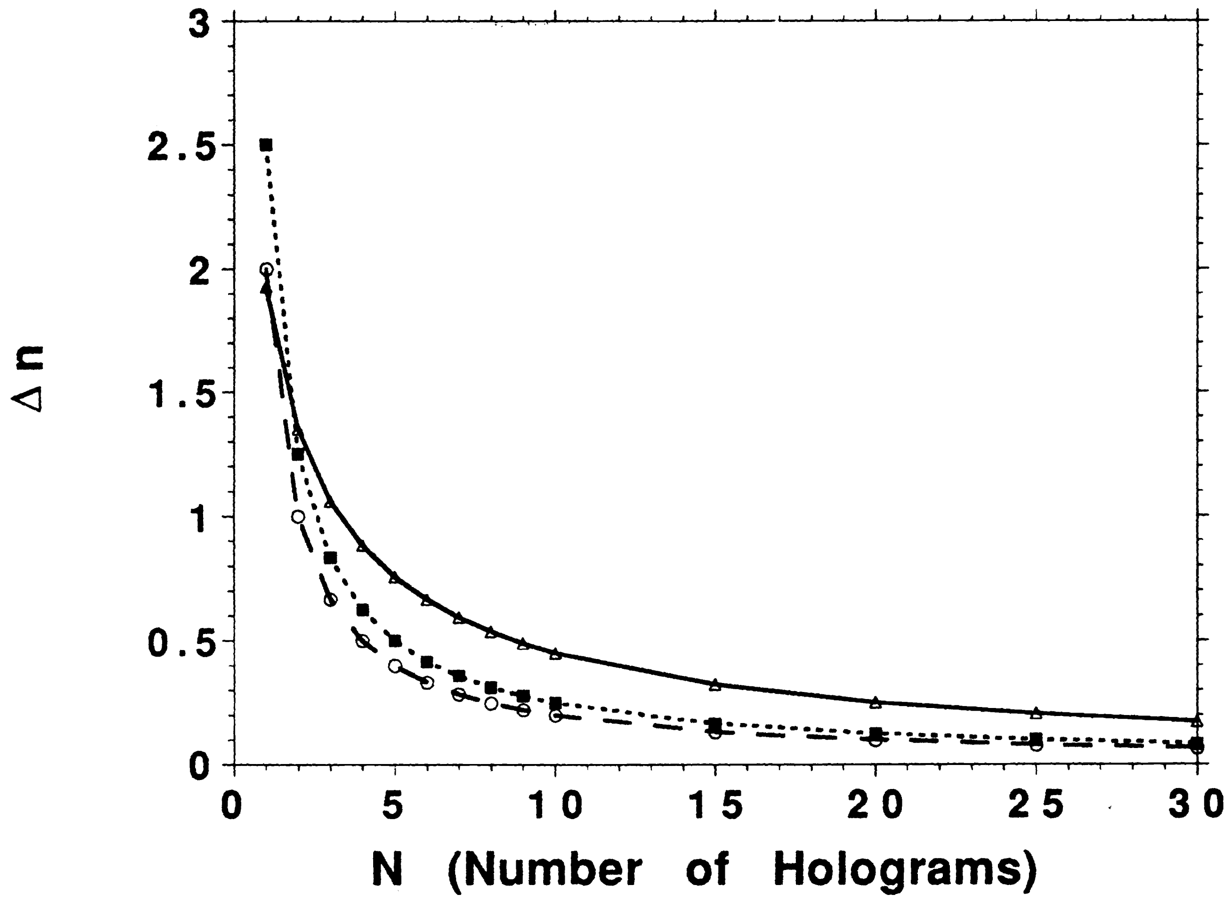

Figure 4: The photorefractive index-perturbation <Δn> in three cases: (I) from the numerical solution of Eqs. 2-4, with amplitude ratio 1:2 (upper solid curve), (II) for uncoupled beams with amplitude ration 1:2 (lower dashed curve) and (III) for uncoupled beams with amplitude ration 1:1 (middle dotted curve)